Chapter 6
Cost-Volume-Profit Relationships

Solutions to Questions

6-1 The contribution margin (CM) ratio is the ratio of the total contribution margin to total sales revenue. It can be used in a variety of ways. For example, the change in total contribution margin from a given change in total sales revenue can be estimated by multiplying the change in total sales revenue by the CM ratio. If fixed costs do not change, then a dollar increase in contribution margin will result in a dollar increase in net operating income. The CM ratio can also be used in break-even analysis. Therefore, knowledge of a product's CM ratio is extremely helpful in forecasting contribution margin and net operating income.

6-2 Incremental analysis focuses on the changes in revenues and costs that will result from a particular action.

6-3 All other things equal, Company B, with its higher fixed costs and lower variable costs, will have a higher contribution margin ratio than Company A. Therefore, it will tend to realize a larger increase in contribution margin and in profits when sales increase.

6-4 Operating leverage measures the impact on net operating income of a given percentage change in sales. The degree of operating leverage at a given level of sales is computed by dividing the contribution margin at that level of sales by the net operating income at that level of sales.

6-5 The break-even point is the level of sales at which profits are zero. It can also be defined as the point where total revenue equals total cost or as the point where total contribution margin equals total fixed cost.

6-6 Three approaches to break-even analysis are (a) the graphical method, (b) the equation method, and (c) the contribution margin method.

In the graphical method, total cost and total revenue data are plotted on a graph. The intersection of the total cost and the total revenue lines indicates the break-even point. The graph shows the break-even point in both units and dollars of sales.

The equation method uses some variation of the equation Sales = Variable expenses + Fixed expenses + Profits, where profits are zero at the break-even point. The equation is solved to determine the break-even point in units or dollar sales.

In the contribution margin method, total fixed cost is divided by the contribution margin per unit to obtain the break-even point in units. Alternatively, total fixed cost can be divided by the contribution margin ratio to obtain the break-even point in sales dollars.

6-7 (a) If the selling price decreased, then the total revenue line would rise less steeply, and the break-even point would occur at a higher unit volume. (b) If the fixed cost increased, then both the fixed cost line and the total cost line would shift upward and the break-even point would occur at a higher unit volume. (c) If the variable cost increased, then the total cost line would rise more steeply and the break-even point would occur at a higher unit volume.

6-8 The margin of safety is the excess of budgeted (or actual) sales over the break-even volume of sales. It states the amount by which sales can drop before losses begin to be incurred.

6-9 The sales mix is the relative proportions in which a company's products are sold. The usual assumption in cost-volume-profit analysis is that the sales mix will not change.

6-10 A higher break-even point and a lower net operating income could result if the sales mix shifted from high contribution margin products to low contribution margin products. Such a shift would cause the average contribution margin ratio in the company to decline, resulting in less total contribution margin for a given amount of sales. Thus, net operating income would decline. With a lower contribution margin ratio, the break-even point would be higher because more sales would be required to cover the same amount of fixed costs.

Exercise 6-1 (20 minutes)

1. The new income statement would be:

	Total	*Per Unit*
Sales (8,050 units)	$209,300	$26.00
Variable expenses	144,900	18.00
Contribution margin	64,400	$ 8.00
Fixed expenses	56,000	
Net operating income	$ 8,400	

You can get the same net operating income using the following approach.

Original net operating income	$8,000
Change in contribution margin (50 units × $8.00 per unit)	400
New net operating income	$8,400

2. The new income statement would be:

	Total	*Per Unit*
Sales (7,950 units)	$206,700	$26.00
Variable expenses	143,100	18.00
Contribution margin	63,600	$ 8.00
Fixed expenses	56,000	
Net operating income	$ 7,600	

You can get the same net operating income using the following approach.

Original net operating income	$8,000
Change in contribution margin (-50 units × $8.00 per unit)	(400)
New net operating income	$7,600

Exercise 6-1 (continued)

3. The new income statement would be:

	Total	*Per Unit*
Sales (7,000 units)	$182,000	$26.00
Variable expenses	126,000	18.00
Contribution margin	56,000	$ 8.00
Fixed expenses	56,000	
Net operating income	$ 0	

Note: This is the company's break-even point.

Exercise 6-2 (20 minutes)

1. The CVP graph can be plotted using the three steps outlined in the text. The graph appears on the next page.

   Step 1. Draw a line parallel to the volume axis to represent the total fixed expense. For this company, the total fixed expense is $12,000.

   Step 2. Choose some volume of sales and plot the point representing total expenses (fixed and variable) at the activity level you have selected. We'll use the sales level of 2,000 units.

Fixed expense	$12,000
Variable expense (2,000 units × $24 per unit)	48,000
Total expense.....	$60,000

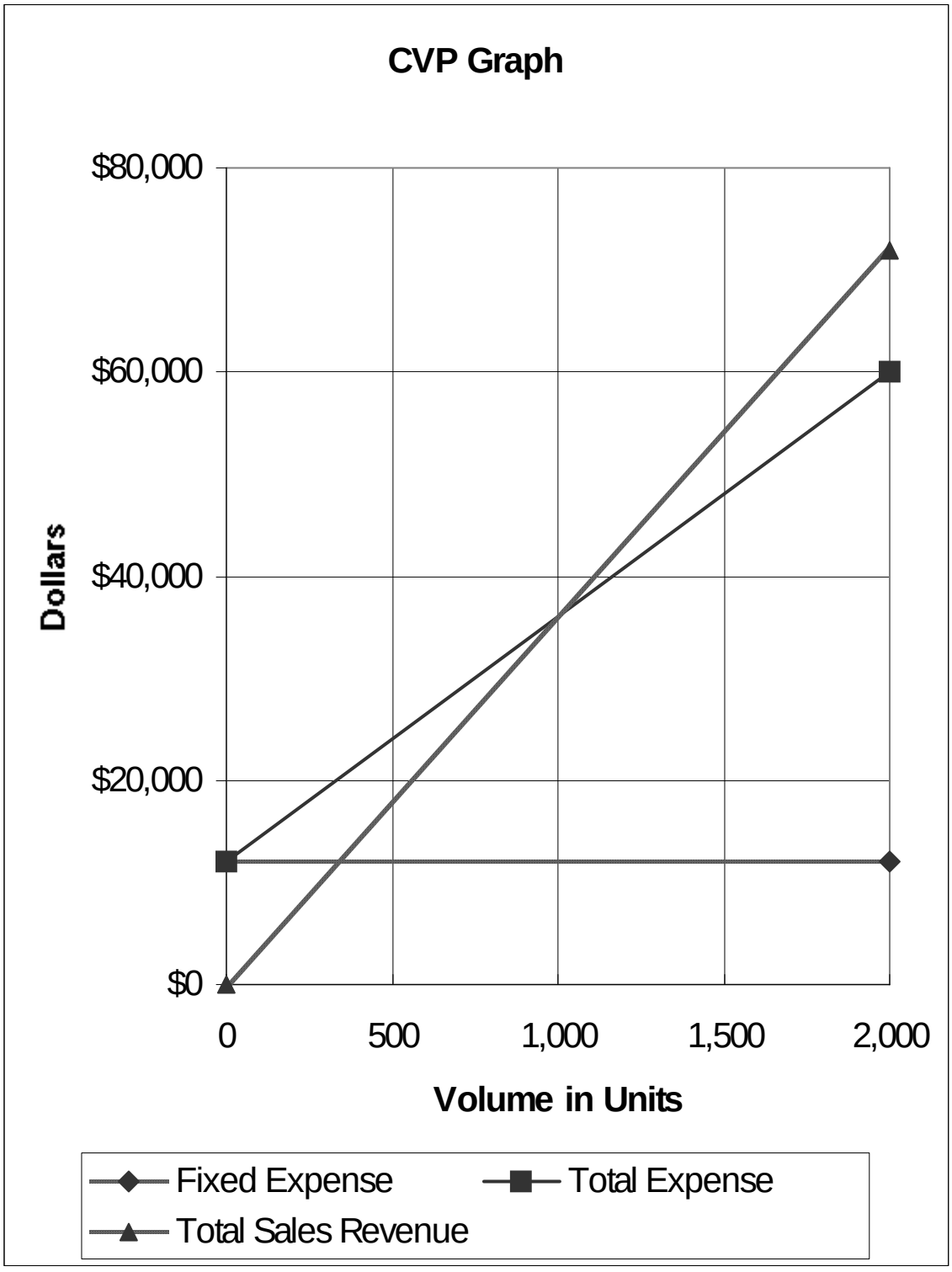
   Step 3. Choose some volume of sales and plot the point representing total sales dollars at the activity level you have selected. We'll use the sales level of 2,000 units again.

Total sales revenue (2,000 units × $36 per unit)...	$72,000

2. The break-even point is the point where the total sales revenue and the total expense lines intersect. This occurs at sales of 1,000 units. This can be verified by solving for the break-even point in unit sales, Q, using the equation method as follows:

   Sales = Variable expenses + Fixed expenses + Profits
   $36Q = $24Q + $12,000 + $0
   $12Q = $12,000
   Q = $12,000 ÷ $12 per unit
   Q = 1,000 units

Exercise 6-2 (continued)



Exercise 6-3 (10 minutes)

1. The company's contribution margin (CM) ratio is:

Total sales	$300,000
Total variable expenses	240,000
= Total contribution margin	60,000
÷ Total sales	$300,000
= CM ratio	20%

2. The change in net operating income from an increase in total sales of $1,500 can be estimated by using the CM ratio as follows:

Change in total sales	$1,500
× CM ratio	20%
= Estimated change in net operating income	$ 300

This computation can be verified as follows:

Total sales	$300,000	
÷ Total units sold	40,000	units
= Selling price per unit	$7.50	per unit

Increase in total sales	$1,500	
÷ Selling price per unit	$7.50	per unit
= Increase in unit sales	200	units
Original total unit sales	40,000	units
New total unit sales	40,200	units

	Original	*New*
Total unit sales	40,000	40,200
Sales	$300,000	$301,500
Variable expenses	240,000	241,200
Contribution margin	60,000	60,300
Fixed expenses	45,000	45,000
Net operating income	$ 15,000	$ 15,300

Exercise 6-4 (20 minutes)

1. The following table shows the effect of the proposed change in monthly advertising budget:

	Current Sales	*Sales With Additional Advertising Budget*	*Difference*
Sales	$225,000	$240,000	$15,000
Variable expenses	135,000	144,000	9,000
Contribution margin	90,000	96,000	6,000
Fixed expenses	75,000	83,000	8,000
Net operating income	$ 15,000	$ 13,000	$(2,000)

Assuming that there are no other important factors to be considered, the increase in the advertising budget should not be approved since it would lead to a decrease in net operating income of $2,000.

Alternative Solution 1

Expected total contribution margin:	
$240,000 × 40% CM ratio	$96,000
Present total contribution margin:	
$225,000 × 40% CM ratio	90,000
Incremental contribution margin	6,000
Change in fixed expenses:	
Less incremental advertising expense	8,000
Change in net operating income	$(2,000)

Alternative Solution 2

Incremental contribution margin:	
$15,000 × 40% CM ratio	$ 6,000
Less incremental advertising expense	8,000
Change in net operating income	$(2,000)

Exercise 6-4 (continued)

2. The $3 increase in variable costs will cause the unit contribution margin to decrease from $30 to $27 with the following impact on net operating income:

Expected total contribution margin with the higher-quality components:	
3,450 units × $27 per unit	$93,150
Present total contribution margin:	
3,000 units × $30 per unit	90,000
Change in total contribution margin.....	$ 3,150

Assuming no change in fixed costs and all other factors remain the same, the higher-quality components should be used.

Exercise 6-5 (20 minutes)

1. The equation method yields the break-even point in unit sales, Q, as follows:

   Sales = Variable expenses + Fixed expenses + Profits
   $8Q = $6Q + $5,500 + $0
   $2Q = $5,500
   Q = $5,500 ÷ $2 per basket
   Q = 2,750 baskets

2. The equation method can be used to compute the break-even point in sales dollars, X, as follows:

	Per Unit	*Percent of Sales*
Sales price.....	$8	100%
Variable expenses	6	75%
Contribution margin	$2	25%

   Sales = Variable expenses + Fixed expenses + Profits
   X = 0.75X + $5,500 + $0
   0.25X = $5,500
   X = $5,500 ÷ 0.25
   X = $22,000

3. The contribution margin method gives an answer that is identical to the equation method for the break-even point in unit sales:

   Break-even point in units sold = Fixed expenses ÷ Unit CM
   = $5,500 ÷ $2 per basket
   = 2,750 baskets

4. The contribution margin method also gives an answer that is identical to the equation method for the break-even point in dollar sales:

   Break-even point in sales dollars = Fixed expenses ÷ CM ratio
   = $5,500 ÷ 0.25
   =$22,000

Exercise 6-14 (30 minutes)

1. Variable expenses: $60 × (100% – 40%) = $36.

2. a.

Selling price.....	$60	100%
Variable expenses.....	36	60%
Contribution margin	$24	40%

Let Q = Break-even point in units.

Sales = Variable expenses + Fixed expenses + Profits
$60Q = $36Q + $360,000 + $0
$24Q = $360,000
Q = $360,000 ÷ $24 per unit
Q = 15,000 units

In sales dollars: 15,000 units × $60 per unit = $900,000

Alternative solution:

Let X = Break-even point in sales dollars.
X = 0.60X + $360,000 + $0
0.40X = $360,000
X = $360,000 ÷ 0.40
X = $900,000

In units: $900,000 ÷ $60 per unit = 15,000 units

b. $60Q = $36Q + $360,000 + $90,000
$24Q = $450,000
Q = $450,000 ÷ $24 per unit
Q = 18,750 units

In sales dollars: 18,750 units × $60 per unit = $1,125,000

Exercise 6–14 (continued)

Alternative solution:

$$X = 0.60X + \$360{,}000 + \$90{,}000$$
$$0.40X = \$450{,}000$$
$$X = \$450{,}000 \div 0.40$$
$$X = \$1{,}125{,}000$$

In units: $1,125,000 ÷ $60 per unit = 18,750 units

c. The company's new cost/revenue relationships will be:

Selling price	$60	100%
Variable expenses ($36 – $3)	33	55%
Contribution margin	$27	45%

$$\$60Q = \$33Q + \$360{,}000 + \$0$$
$$\$27Q = \$360{,}000$$
$$Q = \$360{,}000 \div \$27 \text{ per unit}$$
$$Q = 13{,}333 \text{ units (rounded).}$$

In sales dollars: 13,333 units × $60 per unit = $800,000 (rounded)

Alternative solution:

$$X = 0.55X + \$360{,}000 + \$0$$
$$0.45X = \$360{,}000$$
$$X = \$360{,}000 \div 0.45$$
$$X = \$800{,}000$$

In units: $800,000 ÷ $60 per unit = 13,333 units (rounded)

Exercise 6–14 (continued)

3 a. $$\text{Break-even point in unit sales} = \frac{\text{Fixed expenses}}{\text{Unit contribution margin}}$$

$$= \$360{,}000 \div \$24 \text{ per unit} = 15{,}000 \text{ units}$$

In sales dollars: 15,000 units × $60 per unit = $900,000

Alternative solution:

$$\text{Break-even point in sales dollars} = \frac{\text{Fixed expenses}}{\text{CM ratio}}$$

$$= \$360{,}000 \div 0.40 = \$900{,}000$$

In units: $900,000 ÷ $60 per unit = 15,000 units

b. $$\text{Unit sales to attain target profit} = \frac{\text{Fixed expenses + Target profit}}{\text{Unit contribution margin}}$$

$$= (\$360{,}000 + \$90{,}000) \div \$24 \text{ per unit}$$

$$= 18{,}750 \text{ units}$$

In sales dollars: 18,750 units × $60 per unit = $1,125,000

Alternative solution:

$$\text{Dollar sales to attain target profit} = \frac{\text{Fixed expenses + Target profit}}{\text{CM ratio}}$$

$$= (\$360{,}000 + \$90{,}000) \div 0.40$$

$$= \$1{,}125{,}000$$

In units: $1,125,000 ÷ $60 per unit = 18,750 units

Exercise 6-14 (continued)

c. $$\text{Break-even point in unit sales} = \frac{\text{Fixed expenses}}{\text{Unit contribution margin}}$$

$$= \$360{,}000 \div \$27 \text{ per unit}$$

$$= 13{,}333 \text{ units (rounded)}$$

In sales dollars: 13,333 units × $60 per unit = $800,000 (rounded)

Alternative solution:

$$\text{Break-even point in sales dollars} = \frac{\text{Fixed expenses}}{\text{CM ratio}}$$

$$= \$360{,}000 \div 0.45 = \$800{,}000$$

In units: $800,000 ÷ $60 per unit = 13,333 (rounded)

Exercise 6-16 (30 minutes)

1. Sales = Variable expenses + Fixed expenses + Profits
   $90Q = $63Q + $135,000 + $0
   $27Q = $135,000
   Q = $135,000 ÷ $27 per lantern
   Q = 5,000 lanterns, or at $90 per lantern, $450,000 in sales

   Alternative solution:

$$\text{Break-even point in unit sales} = \frac{\text{Fixed expenses}}{\text{Unit contribution margin}}$$

$$= \frac{\$135{,}000}{\$27 \text{ per lantern}} = 5{,}000 \text{ lanterns,}$$

   or at $90 per lantern, $450,000 in sales

2. An increase in the variable expenses as a percentage of the selling price would result in a higher break-even point. The reason is that if variable expenses increase as a percentage of sales, then the contribution margin will decrease as a percentage of sales. A lower CM ratio would mean that more lanterns would have to be sold to generate enough contribution margin to cover the fixed costs.

Exercise 6-16 (continued)

3.

	Present: 8,000 Lanterns		*Proposed: 10,000 Lanterns**	
	Total	*Per Unit*	*Total*	*Per Unit*
Sales	$720,000	$90	$810,000	$81 **
Variable expenses	504,000	63	630,000	63
Contribution margin	216,000	$27	180,000	$18
Fixed expenses	135,000		135,000	
Net operating income	$ 81,000		$ 45,000	

* 8,000 lanterns × 1.25 = 10,000 lanterns
** $90 per lantern × 0.9 = $81 per lantern

As shown above, a 25% increase in volume is not enough to offset a 10% reduction in the selling price; thus, net operating income decreases.

4. Sales = Variable expenses + Fixed expenses + Profits
$81Q = $63Q + $135,000 + $72,000
$18Q = $207,000
Q = $207,000 ÷ $18 per lantern
Q = 11,500 lanterns

Alternative solution:

$$\text{Unit sales to attain target profit} = \frac{\text{Fixed expenses + Target profit}}{\text{Unit contribution margin}}$$

$$= \frac{\$135{,}000 + \$72{,}000}{\$18 \text{ per lantern}} = 11{,}500 \text{ lanterns}$$

Problem 6-19 (60 minutes)

1. The CM ratio is 30%.

	Total	*Per Unit*	*Percentage*
Sales (13,500 units)	$270,000	$20	100%
Variable expenses	189,000	14	70%
Contribution margin.....	$ 81,000	$ 6	30%

The break-even point is:

Sales = Variable expenses + Fixed expenses + Profits
$20Q = $14Q + $90,000 + $0
$ 6Q = $90,000
Q = $90,000 ÷ $6 per unit
Q = 15,000 units

15,000 units × $20 per unit = $300,000 in sales

Alternative solution:

$$\text{Break-even point in unit sales} = \frac{\text{Fixed expenses}}{\text{Unit contribution margin}}$$

$$= \frac{\$90,000}{\$6 \text{ per unit}} = 15,000 \text{ units}$$

$$\text{Break-even point in sales dollars} = \frac{\text{Fixed expenses}}{\text{CM ratio}}$$

$$= \frac{\$90,000}{0.30} = \$300,000 \text{ in sales}$$

2. Incremental contribution margin:

$70,000 increased sales × 30% CM ratio.....	$21,000
Less increased fixed costs:	
Increased advertising cost.....	8,000
Increase in monthly net operating income.....	$13,000

Since the company presently has a loss of $9,000 per month, if the changes are adopted, the loss will turn into a profit of $4,000 per month.

Problem 6-19 (continued)

3. Sales (27,000 units × $18 per unit*) $486,000
   Variable expenses
   (27,000 units × $14 per unit) 378,000
   Contribution margin 108,000
   Fixed expenses ($90,000 + $35,000)..... 125,000
   Net operating loss..... $(17,000)

   *$20 – ($20 × 0.10) = $18

4. Sales = Variable expenses + Fixed expenses + Profits
   $ 20Q = $14.60Q* + $90,000 + $4,500
   $5.40Q = $94,500
   Q = $94,500 ÷ $5.40 per unit
   Q = 17,500 units

   *$14.00 + $0.60 = $14.60.

   Alternative solution:

$$\text{Unit sales to attain target profit} = \frac{\text{Fixed expenses + Target profit}}{\text{CM per unit}}$$

$$= \frac{\$90{,}000 + \$4{,}500}{\$5.40 \text{ per unit}^{**}}$$

$$= 17{,}500 \text{ units}$$

   **$6.00 – $0.60 = $5.40.

5. a. The new CM ratio would be:

	Per Unit	*Percentage*
Sales.....	$20	100%
Variable expenses	7	35%
Contribution margin.....	$13	65%

Problem 6-19 (continued)

The new break-even point would be:

$$\text{Break-even point in unit sales} = \frac{\text{Fixed expenses}}{\text{Unit contribution margin}}$$

$$= \frac{\$208{,}000}{\$13 \text{ per unit}} = 16{,}000 \text{ units}$$

$$\text{Break-even point in sales dollars} = \frac{\text{Fixed expenses}}{\text{CM ratio}}$$

$$= \frac{\$208{,}000}{0.65} = \$320{,}000 \text{ in sales}$$

b. Comparative income statements follow:

	Not Automated			*Automated*		
	Total	*Per Unit*	*%*	*Total*	*Per Unit*	*%*
Sales (20,000 units) ...	$400,000	$20	100	$400,000	$20	100
Variable expenses	280,000	14	70	140,000	7	35
Contribution margin....	120,000	$ 6	30	260,000	$13	65
Fixed expenses	90,000			208,000		
Net operating income .	$ 30,000			$ 52,000		

Problem 6-19 (continued)

c. Whether or not one would recommend that the company automate its operations depends on how much risk he or she is willing to take, and depends heavily on prospects for future sales. The proposed changes would increase the company's fixed costs and its break-even point. However, the changes would also increase the company's CM ratio (from 30% to 65%). The higher CM ratio means that once the break-even point is reached, profits will increase more rapidly than at present. If 20,000 units are sold next month, for example, the higher CM ratio will generate $22,000 more in profits than if no changes are made.

The greatest risk of automating is that future sales may drop back down to present levels (only 13,500 units per month), and as a result, losses will be even larger than at present due to the company's greater fixed costs. (Note the problem states that sales are erratic from month to month.) In sum, the proposed changes will help the company if sales continue to trend upward in future months; the changes will hurt the company if sales drop back down to or near present levels.

Note to the Instructor: Although it is not asked for in the problem, if time permits you may want to compute the point of indifference between the two alternatives in terms of units sold; i.e., the point where profits will be the same under either alternative. At this point, total revenue will be the same; hence, we include only costs in our equation:

$$\begin{aligned} \text{Let } Q &= \text{Point of indifference in units sold} \\ \$14Q + \$90{,}000 &= \$7Q + \$208{,}000 \\ \$7Q &= \$118{,}000 \\ Q &= \$118{,}000 \div \$7 \text{ per unit} \\ Q &= 16{,}857 \text{ units (rounded)} \end{aligned}$$

If more than 16,857 units are sold, the proposed plan will yield the greatest profit; if less than 16,857 units are sold, the present plan will yield the greatest profit (or the least loss).

Problem 6-24 (30 minutes)

1. The contribution margin per stein would be:

Selling price		$30
Variable expenses:		
Purchase cost of the steins	$15	
Commissions to the student salespersons	6	21
Contribution margin		$ 9

Since there are no fixed costs, the number of unit sales needed to yield the desired $7,200 in profits can be obtained by dividing the target profit by the unit contribution margin:

$$\frac{\text{Target profit}}{\text{Unit contribution margin}} = \frac{\$7{,}200}{\$9 \text{ per stein}} = 800 \text{ steins}$$

800 steins × $30 per stein = $24,000 in total sales

2. Since an order has been placed, there is now a "fixed" cost associated with the purchase price of the steins (i.e., the steins can't be returned). For example, an order of 200 steins requires a "fixed" cost (investment) of $3,000 (200 steins × $15 per stein = $3,000). The variable costs drop to only $6 per stein, and the new contribution margin per stein becomes:

Selling price	$30
Variable expenses (commissions only)	6
Contribution margin	$24

Since the "fixed" cost of $3,000 must be recovered before Mr. Marbury shows any profit, the break-even computation would be:

$$\text{Break-even point in unit sales} = \frac{\text{Fixed expenses}}{\text{Unit contribution margin}} = \frac{\$3{,}000}{\$24 \text{ per stein}} = 125 \text{ steins}$$

125 steins × $30 per stein = $3,750 in total sales

If a quantity other than 200 steins were ordered, the answer would change accordingly.

Problem 6-25 (45 minutes)

1. a.

	Alvaro		*Bazan*		*Total*	
	Euros	*%*	*Euros*	*%*	*Euros*	*%*
Sales	€800	100	€480	100	€1,280	100
Variable expenses	480	60	96	20	576	45
Contribution margin	€320	40	€384	80	704	55
Fixed expenses					660	
Net operating income					€ 44	

b. Break-even sales = Fixed expenses ÷ CM ratio
= €660 ÷ 0.55 = €1,200

Margin of safety in euros = Actual sales - Break-even sales
= €1,280 - €1,200
= €80

Margin of safety percentage = Margin of safety in euros ÷ Actual sales
= €80 ÷ €1,280
= 6.25%

Problem 6-25 (continued)

2. a.

	Alvaro		*Bazan*		*Cano*		*Total*	
	Euros	*%*	*Euros*	*%*	*Euros*	*%*	*Euros*	*%*
Sales	€800	100	€480	100	€320	100	€1,600	100
Variable expenses	480	60	96	20	240	75	816	51
Contribution margin	€320	40	€384	80	€ 80	25	784	49
Fixed expenses							660	
Net operating income							€ 124	

b. Break-even sales = Fixed expenses ÷ CM ratio
= €660 ÷ 0.49
= €1,347 (rounded)

Margin of safety in euros = Actual sales - Break-even sales
= €1,600 - €1,347
= €253

Margin of safety percentage = Margin of safety in euros ÷ Actual sales
= €253 ÷ €1,600
= 15.81%

3. The reason for the increase in the break-even point can be traced to the decrease in the company's average contribution margin ratio when the third product is added. Note from the income statements above that this ratio drops from 55% to 49% with the addition of the third product. This product,

called Cano, has a CM ratio of only 25%, which causes the average contribution margin ratio to fall.

This problem shows the somewhat tenuous nature of break-even analysis when more than one product is involved. The manager must be very careful of his or her assumptions regarding sales mix when making decisions such as adding or deleting products.

It should be pointed out to the president that even though the break-even point is higher with the addition of the third product, the company's margin of safety is also greater. Notice that the margin of safety increases from €80 to €253 or from 6.25% to 15.81%. Thus, the addition of the new product shifts the company much further from its break-even point, even though the break-even point is higher